Chapter 2 First-Order Systems

Sect. 2.1 Modeling via Systems

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Overview

Overview of Chapter 2

In Chapter 2,

- We study systems of DE
- Three techniques: analytic, qualitative and numeric.
- Example: the harmonic oscillator.

Overview of Chapter 2

1. Modeling via Systems
2. The Geometry of Systems
3. The Damped Harmonic Oscillator (Analytic Technique)
4. Additional Analytic Methods for Special Systems (Decoupled Systems)
5. ~~Euler's Method for Systems~~
6. Existence and Uniqueness for Systems
7. ~~The SIR Model of an Epidemic~~
8. ~~The Lorenz Equations~~

The Predator-Prey System Revisited

Consider

$$\frac{dR}{dt} = 2R - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF$$

Equilibrium Solution: (Detail 1)

$$(0, 0), \left(\frac{1}{0.9}, \frac{2}{1.2}\right).$$

([PRG], p.150)

$R(t)$- and $F(t)$- graphs

Now that the equations involve two dependent variables, we assign two initial values

$$R = R_0, \quad F = F_0.$$

Also, now a **solution** of the system means two functions $R(t)$ and $F(t)$ that, taken together, satisfy the system of equations.

$R(t)$- and $F(t)$- graphs

Let $R_0 = 1$ and $F_0 = 0.5$. Euler's method (covered in Sect. 2.5) yields

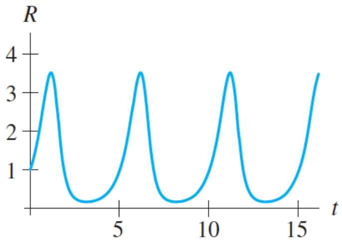


Figure 2.1
The $R(t)$-graph if $R_0 = 1$ and $F_0 = 0.5$.

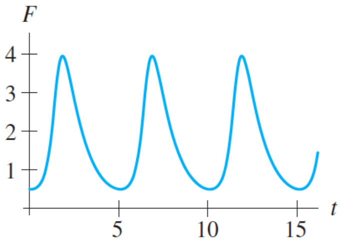
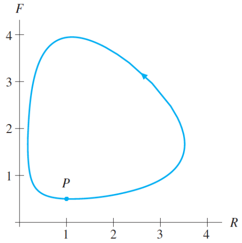


Figure 2.2
The $F(t)$-graph if $R_0 = 1$ and $F_0 = 0.5$.

Question) How do they affect each other? It is hard to see based on these graphs.

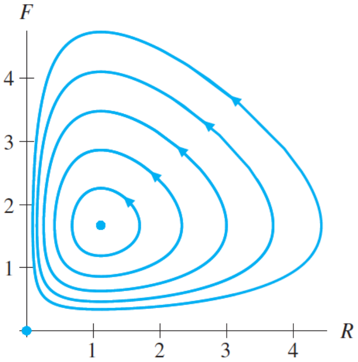
The Phase Portrait for this System

- Given (R_0, F_0) and a solution $R(t), F(t)$, for each t, we can think of $(R(t), F(t))$ as a point in the RF-plane.
- As t varies, the pair $(R(t), F(t))$ sweeps out a curve in the RF-plane.
- This curve is called **solution curve**.



The Phase Portrait for this System

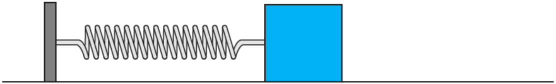
- The *RF*-plane is called the **phase plane**.
- We can draw the **phase portrait**



Now it is clear to see how they influence each other. (Detail 2)

The Motion of a Mass Attached to a Spring

Consider ▸ Harmonic Oscillator



Assumptions

- No friction on table, no air resistance, no damping force.
- The only force acting on the mass is the force of the spring.

([PRG] p. 157)

The Motion of a Mass Attached to a Spring

- Let $y(t)$ denote the position of the mass at time t. Then

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0.$$

- This is called a simple (or undamped) **harmonic oscillator**.
- As it contains a second derivative, it is a **second-order** DE.
- Question) To analyze a second-order DE, can we make it first-order?

([PRG] p. 158)

The Motion of a Mass Attached to a Spring

For

$$\frac{d^2y}{dt} + \frac{k}{m}y = 0$$

if we set $v(t) = dy/dt$, (Detail 3) the equation turns into the first-order system

$$\frac{dy}{dt} = v$$
$$\frac{dv}{dt} = \frac{-k}{m}y.$$

Overview

What's next: 2.2 The Geometry of Systems

homework

- Homework Exercises (required to submit): 19

References

Paul Blanchard, Robert L. Devaney, Glen R. Hall
Differential Equations, fourth edition.