Chapter 2 First-Order Systems

Sect. 2.3 The Damped Harmonic Oscillator(Analytic Technique)

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Overview

The Damped Harmonic Oscillator

In this section, we describe an analytic technique that applies to the damped harmonic oscillator. ▸ Damped Harmonic Oscillator Consider ▸ Suspension in an Automobile 3:30

Considering the damping force, we get (Detail 1)

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0.$$

where $b > 0$ is called **damping coefficient** and the equation is called the **damped harmonic oscillator**.

Question) How can we solve it?

([PRG], p.183)

Guessing Solutions

Consider

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0.$$

We guess $y(t) = e^{st}$. (Detail 2)
Then

$$s^2 + 3s + 2 = 0.$$

Hence

$$y_1(t) = e^{-t}, \quad y_2(t) = e^{-2t}$$

are solutions.
Question: Can we also understand this by using the geometry of the system?
([PRG], p.185)

Guessing Solutions

(Detail 3)

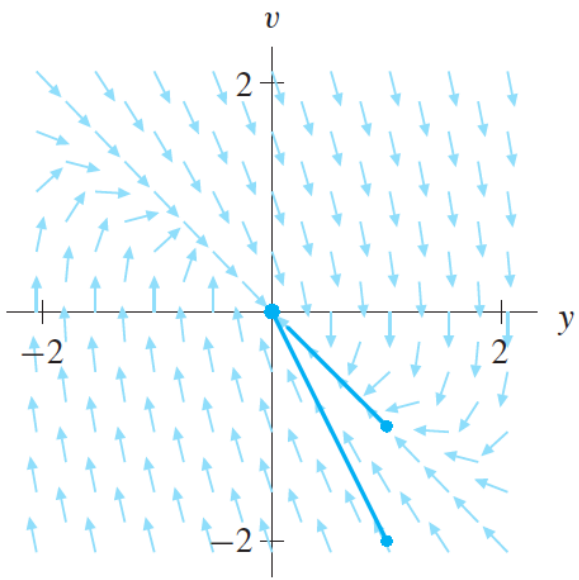

Figure 2.37
The two solution curves that correspond to the solutions $y_1(t) = e^{-t}$ and $y_2(t) = e^{-2t}$. Both curves lie on lines in the yv-plane.

Do we have any idea about how fast they converges to the origin?

Guessing Solutions

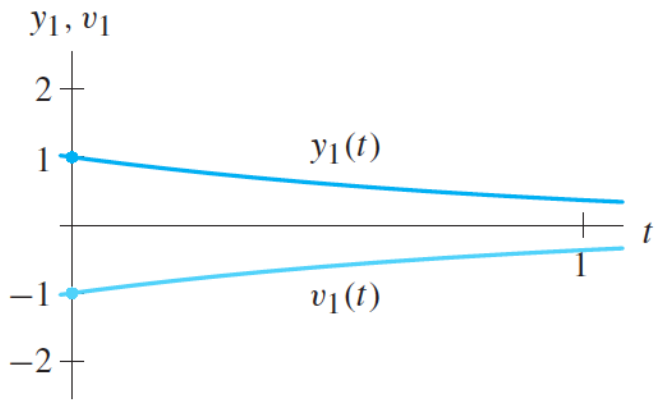

Figure 2.38
The $y(t)$- and $v(t)$-graphs for the solution $y_1(t) = e^{-t}$.

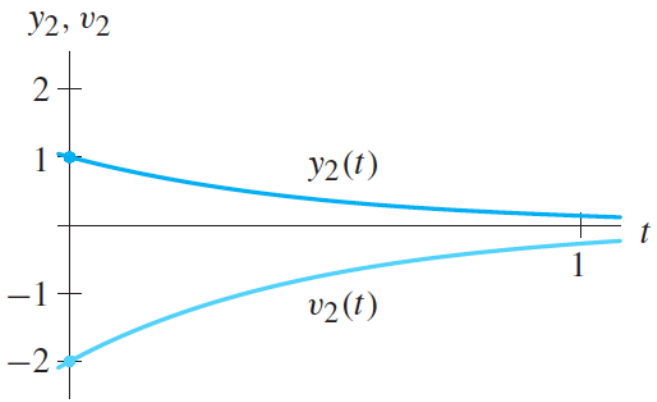

Figure 2.39
The $y(t)$- and $v(t)$-graphs for the solution $y_2(t) = e^{-2t}$.

([PRG], p.186)

How General is this Method?

Questions:

- What kind of systems can we solve using this approach?
- What happens if the roots of the resulting quadratic equations are complex numbers rather than real numbers?

These will be discussed in Chapt. 3 Linear Systems.

Overview

What's next: 2.4 Additional Analytic Methods for Special Systems (Decoupled Systems)

Homework

- Homework Exercises (required to submit): 1(b),(c), 3(b),(c), 7

References


Paul Blanchard, Robert L. Devaney, Glen R. Hall
Differential Equations, fourth edition.