Chapter 2 First-Order Systems

Sect. 2.4 Additional Analytic Methods for Special Systems

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Overview

Additional Analytic Methods for Special Systems

We study analytic techniques that applies to very special classes of systems:

1. the linear systems (in Chapter 3),
2. decoupled systems in this section.

([PRG], p.189)

A Completely Decoupled Example

Consider the system

$$\frac{dx}{dt} = -2x$$
$$\frac{dy}{dt} = -y.$$

We can solve for x, y separately to obtain

$$(x(y), y(t)) = (k_1 e^{-2t}, k_2 e^{-t}).$$

We can understand the solution geometrically. ([PRG], p.191)

A Completely Decoupled Example

We draw the solution curve for $(x(0), y(0)) = (1, 1)$. (Detail 1)

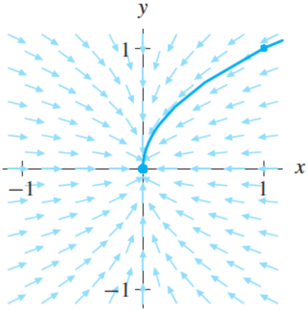

Figure 2.40
The solution curve
$\mathbf{Y}(t) = (e^{-2t}, e^{-t})$.

Question: How fast does it converge to the origin?

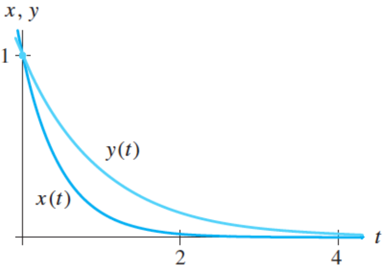

Figure 2.41
The $x(t)$- and $y(t)$-graphs for the solution $(x(t), y(t)) = (e^{-2t}, e^{-t})$.

A partially Decoupled Example

Consider

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = -4y.$$

We first solve for y and then for x to obtain (Detail 2)

$$x(t) = k_1 e^{2t} - \frac{1}{2} k_2 e^{-4t}$$
$$y(t) = k_2 e^{-4t}.$$

We can also understand the solution in a geometric way.

A Partially Decoupled Example

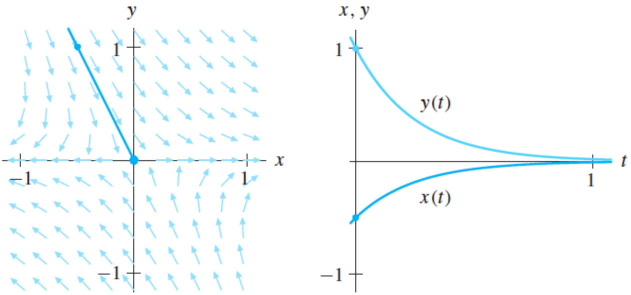

Figure 2.42
Even though the $x(t)$- and $y(t)$-graphs are graphs of exponential functions, the corresponding solution curve lies on a line in the xy-phase plane.

Question: Why do we keep getting a solution curve that lies on a line through the origin?
Do systems always have a solution curve that lies on a line?
It will be answered in Chapt. 3.Linear Systems.

Overview

1. 2.4 Additional Analytic Methods for Special Systems
   - Decoupled Systems
   - Homework

What's next: 2.6 Existence and Uniqueness for Systems

Homework

- Suggested Exercises (optional): 1, 3, 5, 7, 9, 13
- Homework Exercises (required to submit): 3, 7, 9, 13

References

Paul Blanchard, Robert L. Devaney, Glen R. Hall
Differential Equations, fourth edition.