Chapter 4 Forcing and Resonance

Sect. 4.5 The Tacoma Narrows Bridge

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Overview

1. Sect. 4.5 The Tacoma Narrows Bridge
   - Derivation of the Equations
   - Behavior of Solutions
   - Homework

Little Confession

- As you know, in Chapter 4 Forcing and "Resonance", the last section is titled the Tacoma Narrow Bridge.
- So I thought and explained to you that the textbook insists that the collapse of the Tacoma Bridge happened because of resonance.
- However, surprisingly, that is not the way the textbook explains about the Tacoma Bridge.
- Let us see how they actually explain.

The Collapse Is Less Likely Due to Resonance

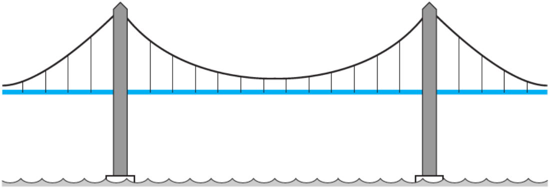

Figure 4.30
Schematic of a suspension bridge.

- The roadbed of a suspension bridge hangs from vertical cables.
- It is very tempting to model the oscillations of the roadbed with a harmonic oscillator equation and to say "Aha, the collapse must be due to resonance".
- However, the wind seldom behaves in such a nice way for very long.
- So, we think differently.

Derivation of the Equations

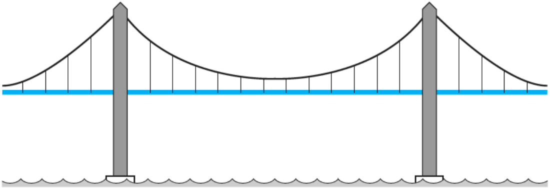

Figure 4.30
Schematic of a suspension bridge.

- The vertical cables act like springs when they are stretched.
- However, when the roadbed is significantly above its rest position, the cables are slack, so they do not push down.

[PRG], p.440

Derivation of the Equations

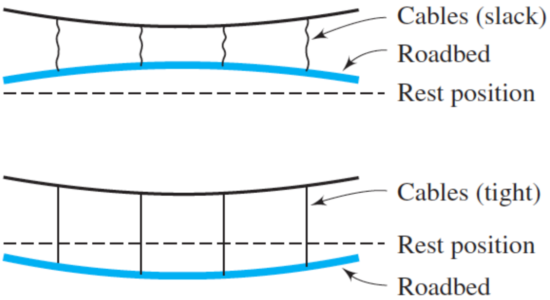

Figure 4.31
Close-up of the vertical cable when the roadbed is above and below the rest position.

Derivation of the Equations

Assumption

The road is 1-dimension. (We don't consider width and thickness.)

Quantity

Let $y(t)$ be the vertical position of the center of the bridge with $y = 0$ corresponding to the position in which the cables taut but not stretched.

Derivation of the Equations

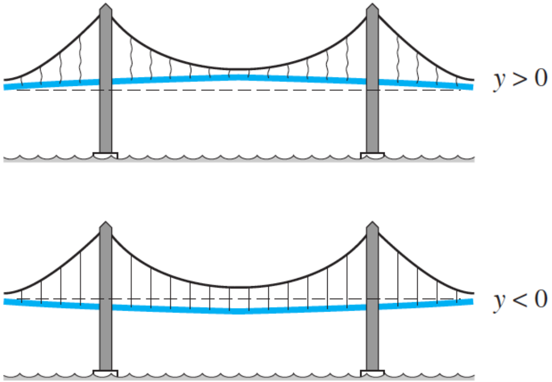

Figure 4.32
Positions of the bridge corresponding to $y > 0$ and $y < 0$.

Derivation of the Equations

We derive an equation to model the Tacoma Bridge:

Equation

$$\frac{d^2y}{dt^2} + \alpha\frac{dy}{dt} + \beta y + c(y) = -g + \lambda \sin \mu t$$

where

- α arises from the damping,
- β accounts for the force provided by the material of the bridge pulling the bridge back toward $y = 0$
- $c(y)$ accounts for the pull of the cable when $y < 0$

$$c(y) = \begin{cases} \gamma y & \text{if } y < 0; \\ 0, & \text{if } y \geq 0. \end{cases}$$

Behavior of Solutions

- To study the equation numerically, we take specific values for the parameters besides λ and get

$$\frac{d^2y}{dt^2} + 0.01\frac{dy}{dt} + h(y) = -10 + \lambda \sin 4t$$

where

$$h(y) = \begin{cases} 17y & \text{if } y < 0; \\ 13y & \text{if } y \geq 0. \end{cases}$$

- We consider two cases separately: 1) λ is small (< 0.05), 2) λ is big.

([PRG], p. 443)

Behavior of Solutions

Recall

> *a forced harmonic oscillator with damping and sinusoidal forcing has one periodic solution to which every other solution tends as time increases, the steady-state solution.*

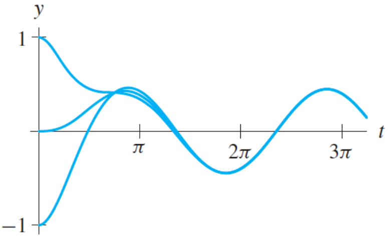

Figure 4.9
Several solutions of the equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \sin t.$$

Behavior of Solutions

When λ is small, every solution tends toward a periodic solution with small magnitude near $y = -10/17$ as usual, but...

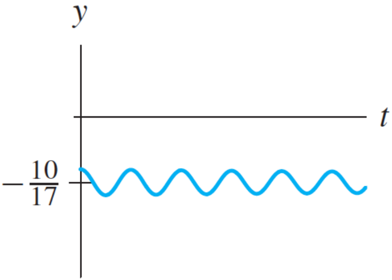

Figure 4.34
Solution of the system with small forcing.

Behavior of Solutions

As λ increases, a new phenomenon is observed.

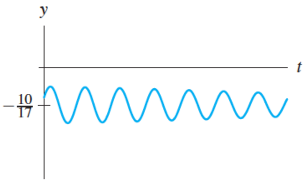

Figure 4.36
Solution of the forced system with larger forcing than in Figure 4.34 and initial conditions near the equilibrium.

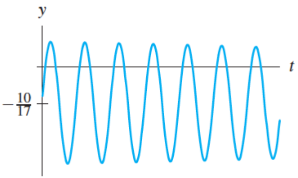

Figure 4.37
Solution of the forced system with the same large forcing as in Figure 4.36 but with initial conditions farther from the equilibrium.

Behavior of Solutions

Conclusion

Therefore, a single strong gust could suddenly cause the bridge to begin oscillating with much larger amplitude perhaps with devastating consequences.

Overview of Sect. 4.5 The Tacoma Narrows Bridge

What's next: Chapt. 5 Nonlinear Systems

Homework

- No homework.

References

Paul Blanchard, Robert L. Devaney, Glen R. Hall
Differential Equations, fourth edition.